MTH 175 Calculus / Knick
1.2 Formal delta-epsilon proof #4

24/25

Show using a complete proof that:

$$\lim_{x \to 1} \left(\frac{2x^2 + 2x - 4}{x - 1} \right) = 6 \Rightarrow \frac{(2x-2)(x+2)}{x-1} \Rightarrow \frac{2\cancel{(x-1)}(x+2)}{\cancel{(x-1)}} \Rightarrow 2x+4$$

Let $\varepsilon > 0$. $\exists\, \delta > 0 \ni \quad 0 < |x - c| < \delta \Leftrightarrow |f(x) - L| < \varepsilon$

$$0 < |x-1| < \delta \Leftrightarrow \left| \left(\frac{2x^2+2x-4}{x-1} \right) - 6 \right| < \varepsilon$$

$$x \neq 1,\ -\delta < x - 1 < \delta$$

$$1 - \delta < x < \delta + 1$$

$$\Rightarrow x \in (1-\delta, \delta+1)$$

→ need another step

$$\left| \frac{2x^2+2x-4-6x+6}{x-1} \right| < \varepsilon$$

$$\left| \frac{2x^2-4x+2}{(x-1)} \right| < \varepsilon$$

$$\left| \frac{(2x-2)\cancel{(x-1)}}{\cancel{(x-1)}} \right| < \varepsilon$$

$$|2x-2| < \varepsilon,\ x \neq 1$$

$$|2(x-1)| < \varepsilon,\ x \neq 1$$

$$|2| \cdot |x-1| < \varepsilon,\ x \neq 1$$

$$2|x-1| < \varepsilon,\ x \neq 1 \rightarrow |x-1| < \varepsilon/2$$

$$\therefore \delta \leq \varepsilon/2,\ x \neq 1$$

[need connect statem't]

Since $|x-1| < \delta$ and $|x-1| < \varepsilon/2$ Take $\delta = \varepsilon/2$ noting $\delta < \varepsilon/2$ also works.

Thus, $-\delta < x - 1 < \delta$, but $\delta = \varepsilon/2$, $x \neq 1$

$$-\varepsilon/2 < x - 1 < \varepsilon/2,\ x \neq 1$$

$$-\varepsilon < 2(x-1) < \varepsilon,\ x \neq 1$$

$$-\varepsilon < 2x - 2 < \varepsilon,\ x \neq 1$$

$$-\varepsilon < \frac{(2x-2)(x-1)}{(x-1)} < \varepsilon,\ x \neq 1$$

$$-\varepsilon < \frac{2x^2-4x+2}{x-1} < \varepsilon,\ x \neq 1$$

$$-\varepsilon < \frac{2x^2+(-6x+2x)+(6-4)}{x-1} < \varepsilon,\ x \neq 1$$

$$-\varepsilon < \frac{2x^2+2x-4-6x+6}{x-1} < \varepsilon,\ x \neq 1$$

$$-\varepsilon < \frac{2x^2+2x-4-6(x-1)}{(x-1)} < \varepsilon,\ x \neq 1$$

$$-\varepsilon < \frac{2x^2+2x-4}{(x-1)} - 6 < \varepsilon,\ x \neq 1$$

$$\left| \left(\frac{2x^2+2x-4}{x-1} \right) - 6 \right| < \varepsilon,\ x \neq 1$$

$$\boxed{\therefore \lim_{x \to 1} \left(\frac{2x^2+2x-4}{x-1} \right) = 6 \quad \forall\, \varepsilon > 0,\ x \neq 1.}$$

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$$\lim_{x\to 1}\left(\frac{2x^2+2x-4}{x-1}\right)=6,\ x\neq 1 \Rightarrow \lim_{x\to 1} 2x+4=6,\ x\neq 1$$